Stochastic

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→ Sheet 2 ←

Distributive Law:-

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Demorgan's Law

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

Theorem 3:-

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

Ex: prove that $p(A^c) = 1 - p(A)$

$$A^c \cup A = S$$
$$P(A^c \cup A) = p(S)$$
$$p(A) + p(A^c) - \underbrace{p(A^c \cap A)}_{0} = 1$$
$$p(A^c) = 1 - p(A)$$

Q1: a) A but not B $\Rightarrow A - B,\ A - (A \cap B)$

b) Either A or B but not both

$$\Rightarrow (A \cup B) - (A \cap B)$$
$$,\ (A - B) \cup (B - A)$$

b) A or not b

$$\Rightarrow A \cup B^c,\ (B - A)^c$$

Q2: a) A and B but not C

$$\Rightarrow (A \cap B) - C$$

b) only A

$$\Rightarrow A - (B \cup C)$$

Q3: Coin and Die

a) A: head and even number

B: prime number

C: tails and odd no.

$S = \{(H,1), (H,2) \cdots (t,1), (t,2) \cdots\}$

$A = \{(H,2), (H,4), (H,6)\}$

$B = \{(H,2), (H,3), (H,5), (t,2), (t,3), (t,5)\}$

$C = \{(T,1), (T,3), (T,5)\}$

b) A or B

$\{(H,2)(H,3)(H,4), (H,5)(H,6)(T,2)(T,3)(T,5)\}$

c) only B

$B - (A \cup C) = (B - A) \cup (B - C)$

$= \{(H,3)(H,5)(T,2)\}$

d) $P(H) = 2\,P(T)$

$P(H) + P(T) = 1$

$2\,P(T) + P(T) = 1 \Rightarrow P(T) = 1/3$

$P(H) = 2/3$

Q7:- 10 boys, 20 girls

half boys and half girls have brown eyes

$P(\text{a boy or has brown eyes}) = P(E_1 \cup E_2)$

E_1: a boy

$P(E_1) = 1/3$

E_2: brown eyes

$P(E_2) = 5/30 = 1/6$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= 1/3 + 1/2 - 1/6$

$= 2/3$

Theorem

$S = \{ (c_1, c_2, c_3 \ldots), (c_1, c_2, c_3, \ldots \}$

$P(\text{sample point}) = P(c_1)\,P(c_2)\,P(c_3) \ldots$

Q10: The odds that an event will occuer are a:b

$$\frac{a}{a+b}$$

Q11: 3 to 2

$P(\text{Event}) = \frac{3}{5}$;

Q 18: Die, Probability of a number appearing is proportional to the number

$P(1) = 1/21$, $P(2) = 2/21$

$P(3) = 3/21$

A: even number

$P(A) = \frac{12}{21} = \frac{2}{21} + \frac{4}{21} + \frac{6}{21}$

Q 19: Prove that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$P(A \cup (B \cup C)) = P(A) + P(B \cup C) - P(A \cap (B \cup C))$

$= P(A) + P(B) + P(C) - P(B \cap C) - \underline{P((A \cap B) \cup (A \cap C))}$

$\Downarrow$

$- [P(A \cap B) + P(A \cap C) - P(A \cap B \cap \cancel{A} \cap C)]$

$P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

5, 6, 8, 12, 13, 15, 20 $\Rightarrow$ Report

* Conditional probability:-

the conditional probability of an event A given that B has occurred is denoted by.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Sheet 3:-

(4) 25 % failed in math
15 % failed in chemistry
10 % failed in both

a) if he failed in chemistry, what is the probability that he failed in math.

E_1 : Math , $P(E_1) = 0.25$

E_2 : Chem. , $P(E_2) = 0.15$

$P(E_1 \cap E_2) = 0.1$

$$P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{0.1}{0.15} = \frac{10}{15} = \frac{2}{3}$$

b) if he failed in math, what is the probability that he failed in chemistry

$$P(E_2|E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = 2/5$$

⇒ Turn over

c) fail in math or chem.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= 0.25 + 0.15 - 0.1 = 0.3$$

Q5: Let A and B

$P(A) = 1/2$, $P(B) = 1/3$, $P(A \cap B) = 1/4$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 3/4$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 7/12$$

$$P(A^c | B^c) = \frac{P(A^c \cap B^c)}{P(B^c)} \Rightarrow P(A \cup B)^c = 1 - \frac{7}{12} = 5/12$$

$$= \frac{5/12}{2/3} = 5/8$$

Q6: $P(A) = 3/8$, $P(B) = 5/8$

$P(A \cup B) = 3/4$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{5/8} = 2/3$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= 3/8 + 5/8 - 3/4 = 1/4$$

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* Independent events:-

$$P(B) = P(B|A)$$

Ex: if A, B are independent

- $P(A \cap B) = P(A)\, P(B)$
- $P(B|A) = P(B)$
- $\dfrac{P(A \cap B)}{P(A)} = P(B$

Q1: if A, B are independent events, prove that A^c and B^c are independent.

$$P(A \cap B) = P(A)\, P(B)$$

$$\begin{aligned}
P(A^c \cap B^c) &= P(A \cup B)^c \\
&= 1 - P(A \cup B) \\
&= 1 - P(A) - P(B) + P(A \cap B) \\
&= \underbrace{1 - P(A)} - P(B) + P(A)\, P(B) \\
&= 1 - P(A) - P(B)(1 - P(A)) \\
&= (1 - P(A))(1 - P(B)) \\
&= P(A^c)\, P(B^c)
\end{aligned}$$

Same problem for A, B^c (Prove)

$$\boxed{P(A \cap B^c) = P(A)\,P(B^c)}$$ ← المطلوب

$$P(A \cap B^c) = P(A) - P(A \cap B)$$
$$= P(A) - P(A)P(B)$$
$$= P(A)(1 - P(B))$$
$$= P(A)P(B^c)$$

Q2: 3 fair coins, Find the probability that they are all heads if:

a) the first coin is head

b) one of the coins is head

$S = \{(h,h,h),(h,h,t),\dots\}$

E_1: all heads

$P(E_1) = 1/8$

E_2: first head

$P(E_2) = 1/2$

$$P(E_1 | E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1/8}{1/2} = 1/4$$

E_3: one is head

$P(E_3) = 7/8$, $P(E_1 \cap E_2) = 1/8$

$$P(E_1 | E_3) = \frac{P(E_1 \cap E_2)}{P(E_3)} = \frac{1/8}{7/8} = 1/7$$